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$S_{\tau_{ij}}$ -sag*-Closed Sets in Supra Bitopological Spaces

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ABSTRACT: In this paper, we introduce supra strongly ag^* -closed sets(briefly, $S_{\tau_{ij}}$ -s ag^* -closed sets) and $S_{\tau_{ij}}$ -s ag^* -open sets in supra bitopological spaces and their properties are analyzed. Further, we define and study three supra bitopologicalspaces $S_{\tau_{ij}}$ -s T_g , $S_{\tau_{ij}}$ -s T_g and $S_{\tau_{ij}}$ -s T_g *-spaces. Also, some of their properties have been investigated.

MATHEMATICS SUBJECT CLASSIFICATION: 54D05, 54D10, 54D08, 54D20

KEYWORDS: Supra bitopology, $S_{\tau_{ij}}$ -sag*-closed sets, $S_{\tau_{ij}}$ -sag*-open sets, $S_{\tau_{ij}}$ -s* T_c -space, $S_{\tau_{ij}}$ -s* T_g -space and $S_{\tau_{ij}}$ -s* T_g *-space.

I. INTRODUCTION

In 1963, Kelly[6] initiated the study of bitopological spaces. Sheik John and Sundaram [15] introduced the concept of strongly alpha generalized star closed sets in bitopological spaces. The concept of supra topological spaces have been introduced by Mashhour[10] in 1983. In topological space the arbitrary union condition is enough to have a supra topological space. Gowri and Rajayal[2] are initiated the concept of supra bitopological spaces. The aim of this communication is to introduce the concepts of $S_{\tau_{ij}}$ -sag*-closed sets as well as $S_{\tau_{ij}}$ -sag*-open sets and discussed some of their properties in supra bitopological spaces. Also, three new supra bitopological spaces are introduced namely, $S_{\tau_{ij}}$ -s* T_g and $S_{\tau_{ij}}$ -s* T_g *-spaces. Finally, we examine the characterizations of these types of spaces.

II. PRELIMINARIES

Definition 2.1 [10] (X, S_{τ}) is said to be a supra topological space if it is satisfying these conditions:

(1) $X, \emptyset \in S_{\tau}$

(2) The union of any number of sets in S_{τ} belongs to S_{τ} .

Definition 2.2 [10] Each element $A \in S_{\tau}$ is called a supra open set in (X, S_{τ}) , and its compliment is called a supra closed set in (X, S_{τ}) .

Definition 2.3 [10] If (X,S_{τ}) is a supra topological spaces, $A \subseteq X$, $A \neq \emptyset$, S_{τ_A} is the class of all intersection of A with each element in S_{τ} , then (A,S_{τ_A}) is called a supra subspace of (X,S_{τ}) .

Definition 2.4 [10] The supra closure of the set A is denoted by S_{τ} -cl(A) and is defined as S_{τ} -cl(A) = $\cap \{B : B \text{ is a supra closed and } A \subseteq B\}$.



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Definition 2.5 [10] The supra interior of the set A is denoted by S_{τ} -int(A) and is defined as S_{τ} -int(A) = $\bigcup \{B : B \text{ is a supra open and } B \subseteq A\}$.

Definition 2.6 [2] If S_{τ_1} and S_{τ_2} are two supra topologies on a non-empty set X, then the triplet $(X, S_{\tau_1}, S_{\tau_2})$ is said to be a supra bitopological space.

Definition 2.7 [2] Each element of S_{τ_i} is called a supra τ_i -open sets(briefly, S_{τ_i} -open sets) in $(X, S_{\tau_1}, S_{\tau_2})$. Then the complement of S_{τ_i} -open sets are called a supra τ_i -closed sets(briefly, S_{τ_i} -closed sets), for i = 1, 2.

Definition 2.8 [2] If $(X, S_{\tau_1}, S_{\tau_2})$ is a supra bitopological space, $Y \subseteq X$, $Y \neq \emptyset$, then $(Y, S_{\tau_1^*}, S_{\tau_2^*})$ is a supra bitopological subspace of $(X, S_{\tau_1}, S_{\tau_2})$ if $S_{\tau_1^*} = \{U \cap Y; U \text{ is a } S_{\tau_1^-} \text{ open in } X\}$ and $S_{\tau_2^*} = \{V \cap Y; V \text{ is a } S_{\tau_2^-} \text{ open in } X\}$.

Definition 2.9 [2] The S_{τ_i} -closure of the set A is denoted by S_{τ_i} -cl(A) and is defined as S_{τ_i} -cl(A) = $\cap \{B : B \text{ is a } S_{\tau_i}$ -closed and $A \subseteq B$, for $i = 1, 2\}$.

Definition 2.10 [2] The S_{τ_i} -interior of the set A is denoted by S_{τ_i} -int(A) and is defined as S_{τ_i} -int(A) = $\cup \{B : B \text{ is a } S_{\tau_i}$ -open and $B \subseteq A$, for $i = 1, 2\}$.

Definition 2.11 [3] A subset A of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is called an $S_{\tau_{ij}}$ - α -open if $A \subseteq S_{\tau_i}$ -int $(S_{\tau_j}$ - $cl(S_{\tau_i}$ -int(A))), where $i \neq j$, i, j = 1, 2.

The complement of $S_{\tau_{ii}}$ - α -open set is said to be $S_{\tau_{ii}}$ - α -closed set.

The family of $S_{\tau_{ij}}$ - α -open(resp. $S_{\tau_{ij}}$ - α -closed) sets of X is denoted by $S_{\tau_{ij}}$ - $\alpha O(X)$ (resp. $S_{\tau_{ij}}$ - $\alpha C(X)$), where $i \neq j$, i, j = 1, 2.

Through out this paper, For a subset A of $(X, S_{\tau_1}, S_{\tau_2})$, S_{τ_j} -acl(A) denote the aclosure of A with respect to the topology S_{τ_i} .

Definition 2.12 [4] A subset A of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is said to be supra τ_{ij} -generalized closed(briefly $S_{\tau_{ij}}$ -generalized) if S_{τ_j} -cl(A) $\subseteq U$ whenever $A \subseteq U$ and $U \in S_{\tau_i}$ -open, where i, j = 1, 2 and $i \neq j$.

The complement of $S_{\tau_{ij}}$ -g-closed set is said to be $S_{\tau_{ij}}$ -g- open.

The family of all $S_{\tau_{ij}}$ -g-closed(resp. $S_{\tau_{ij}}$ -g-open)sets of $(X, S_{\tau_1}, S_{\tau_2})$ is denoted by $S_{\tau_{ij}}$ -gC(X)(resp. $S_{\tau_{ij}}$ -gO(X)), where i, j = 1, 2 and $i \neq j$.

Definition 2.13 [5] A subset A of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is said to be supra τ_{ij} -generalized starclosed(briefly, $S_{\tau_{ij}}$ -g*-closed) if S_{τ_i} -cl(A) $\subseteq U$ whenever A $\subseteq U$ and $U \in S_{\tau_i}$ -g-open, where i, j = 1, 2 and $i \neq j$.

The complement of $S_{\tau_{ij}}$ -g*-closed set is said to be $S_{\tau_{ij}}$ -g*- open.

The family of all $S_{\tau_{ij}}$ - g^* -closed(resp. $S_{\tau_{ij}}$ - g^* -open)sets of $(X, S_{\tau_1}, S_{\tau_2})$ is denoted by $S_{\tau_{ij}}$ - $g^*C(X)$ (resp. $S_{\tau_{ij}}$ - $g^*O(X)$), where i, j = 1, 2 and $i \neq j$.

Definition 2.14 [4&5] supra biopological space $(X, S_{\tau_1}, S_{\tau_2})$ is said to be



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- (1) $S_{\tau_{ij}}$ - $T_{1/2}$ -space[4] if every $S_{\tau_{ij}}$ -g-closed set is S_{τ_i} closed.
- (2) Pairwise S_{τ} - $T_{1/2}$ -space[4] if it is $S_{\tau_{11}}$ - $T_{1/2}$ -space and $S_{\tau_{11}}$ - $T_{1/2}$ -space.
- (3) $S_{\tau_{ij}}$ - $T^*_{1/2}$ -space[5] if every $S_{\tau_{ij}}$ -g*-closed set is S_{τ_j} closed.
- (4) Pairwise S_{τ} - $T_{1/2}^*$ -space[5] if it is $S_{\tau_{ij}}$ - $T_{1/2}^*$ -space and $S_{\tau_{ji}}$ - $T_{1/2}^*$ -space.
- (5) $S_{\tau_{ij}}$ - $T_{1/2}^{**}$ -space[5] if every $S_{\tau_{ij}}$ -g-closed set is $S_{\tau_{ij}}$ -g*- closed.

III. $S_{\tau_{ii}}$ -STRONGLY αg^* -CLOSED SETS

In this section we introduce the concept of $S_{\tau_{ii}}$ -strongly αg^* -closed sets in supra bitopological spaces.

Definition 3.1 A subset A of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is said to be supra τ_{ij} -strongly αg^* -closed(briefly, $S_{\tau_{ii}}$ -sag*-closed) if S_{τ_i} - $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in S_{\tau_i}$ - g^* -open, where i, j = 1, 2 and $i \neq j$.

The complement of $S_{\tau_{ij}}$ -sag*-closed set is said to be $S_{\tau_{ij}}$ -sag*- open.

The family of all $S_{\tau_{ij}}$ -sag*-closed(resp. $S_{\tau_{ij}}$ -sag*-open)sets of $(X, S_{\tau_1}, S_{\tau_2})$ is denoted by $S_{\tau_{ij}}$ -s*C(X)(resp. $S_{\tau_{ij}}$ -s*O(X)), where i, j = 1, 2 and $i \neq j$.

Example 3.2 *Let* $X = \{a, b, c\},\$

 $S_{\tau_1} = \{\emptyset, X, \{a, b\}, \{b, c\}\},\$

 $S_{\tau_2} = \{\emptyset, X, \{a, c\}, \{b, c\}\},\$

 $S_{\tau_{12}} - s^* C(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, c\}\}.$

Therefore $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$ -sag*-closed set.

Definition 3.3 A subset A of supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is said to be pairwise S_{τ} -sag*-closed(briefly, p- S_{τ} -sag*-closed) if A is $S_{\tau_{12}}$ -sag*-closed and $S_{\tau_{21}}$ -sag*-closed. The complement of a pairwise S_{τ} -sag*-closed set is said to be pairwise S_{τ} -sag*-open(briefly, p- S_{τ} -sag*-open.)

Remark 3.4 By setting $S_{\tau_1} = S_{\tau_2}$ in Definition 3.1, a $S_{\tau_{11}}$ -sag*-closed becomes

 S_{τ} -sag*- closed.

Proposition 3.5 If A is S_{τ_i} -closed subset of $(X, S_{\tau_1}, S_{\tau_2})$ then A is $S_{\tau_{ii}}$ -sag*closed.

Proof: It is obvious that every S_{τ_i} -closed set is $S_{\tau_{ii}}$ -sag*- closed.

The converse of the above Proposition 3.5, is not true as seen from the following example.

Example 3.6 *Let* $X = \{a, b, c\}$,

 $S_{\tau_1} = \{\emptyset, X, \{a, b\}, \{b, c\}\},\$

 $S_{\tau_2} = \{\emptyset, X, \{a, c\}, \{b, c\}\},\$

$$S_{\tau_{12}} - s^* C(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, c\}\}.$$

Then the subset $\{b\}$ is $S_{\tau_{12}}$ -sag*-closed but it is not S_{τ_2} -closed in $(X, S_{\tau_1}, S_{\tau_2})$.



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Proposition 3.7 If A is S_{τ_i} -g*-closed subset of $(X, S_{\tau_1}, S_{\tau_2})$ then A is $S_{\tau_{ii}}$ -sag*-closed set.

The converse of the above Proposition 3.7, is not true from the following example.

Example 3.8 *Let* $X = \{a, b, c\},\$

 $S_{\tau_1} = \{\emptyset, X, \{a, b\}, \{b, c\}\},\$

 $S_{\tau_2} = \{\emptyset, X, \{c\}, \{a, b\}\},\$

 $S_{\tau_2} - g^* C(X) = \{ \emptyset, X, \{c\}, \{a, b\} \},\$

 $S_{\tau_{12}} - s^* C(X) = \{ \emptyset, X, \{a\}, \{c\}, \{a, b\}, \{a, c\} \}.$

Here the subset $\{a,c\}$ is $S_{\tau_{12}}$ -sag*-closed but it is not S_{τ_2} -g*-closed set in $(X, S_{\tau_1}, S_{\tau_2})$.

Proposition 3.9 If A is both S_{τ_i} -g*-open and $S_{\tau_{ij}}$ -sag*-closed, then A is S_{τ_j} -a-closed.

Proof: Suppose that A is S_{τ_i} - g^* -open and $S_{\tau_{ij}}$ - sag^* -closed. Then $A \subseteq A$ implies that S_{τ_j} - $acl(A) \subseteq A$. Obviously, $A \subseteq S_{\tau_j}$ -acl(A). Therefore A is S_{τ_i} -a-closed.

Proposition 3.10 If A is $S_{\tau_{1i}}$ -g*-closed subset of $(X, S_{\tau_1}, S_{\tau_2})$ then A is $S_{\tau_{1i}}$ -sag*-closed set.

The reverse of the Proposition 3.10, is not true from the following example.

Example 3.11 *Let* $X = \{a, b, c, d\}$ *,*

 $S_{\tau_1} = \{ \emptyset, X, \{a\}, \{a, d\}, \{b, c, d\} \},\$

 $S_{\tau_2} = \{ \emptyset, X, \{b\}, \{b, c\}, \{a, c, d\} \},$

 $S_{\tau_{12}} - g^* C(X) = \{ \emptyset, X, \{b\}, \{a, d\}, \{a, c, d\}, \{a, b, c\} \},\$

 $S_{\tau_{12}} - s^* C(X) = \{ \emptyset, X, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\} \}.$

Then the subset $\{a,b,d\}$ is $S_{\tau_{12}}$ -sag*-closed set but it is not $S_{\tau_{12}}$ -g*-closed set in

 $(X, S_{\tau_1}, S_{\tau_2})$

Remark 3.12 If $A, B \in S_{\tau_{12}}$ -s*C(X) then $A \cup B \notin S_{\tau_{12}}$ -s*C(X). The following example supports our claim.

Example 3.13 *Let* $X = \{a, b, c, d\}$ *,*

 $S_{\tau_1} = \{ \emptyset, X, \{b\}, \{b, c\}, \{a, c, d\} \},\$

 $S_{\tau_2} = \{ \emptyset, X, \{a\}, \{a, d\}, \{b, c, d\} \},\$

 $S_{\tau_{12}} \text{-} s^* C(X) = \{ \emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\} \}.$

Here $A = \{a\}$ and $B = \{c\}$ then $A \cup B = \{a\} \cup \{c\} = \{a, c\}$ not in $S_{\tau_{12}}$ -s*C(X).

Remark 3.14 If $A, B \in S_{\tau_{12}}$ -s^{*}C(X) then $A \cap B \notin S_{\tau_{12}}$ -s^{*}C(X), as seen from the following example.

Example 3.15 In Example 3.13,

 $S_{\tau_{12}} - s^* C(X) = \{ \emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\} \}.$

Here $A = \{a, b, d\}$ and $B = \{b, c, d\}$ then $A \cap B = \{a, b, d\} \cap \{b, c, d\} = \{b, d\}$ not in $S_{\tau_{12}}$ -s*C(X).

Remark 3.16 $S_{\tau_{12}}$ -s*C(X) is generally not equal to $S_{\tau_{21}}$ -s*C(X). Then the following example supports our claim.



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Example 3.17 *Let* $X = \{a, b, c\},\$

 $S_{\tau_1} = \{\emptyset, X, \{c\}, \{a, b\}, \{b, c\}\},\$

 $S_{\tau_2} = \{\emptyset, X, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\},\$

 $S_{\tau_{12}}\text{-}s^*C(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}\} \text{ and }$

 $S_{\tau_{21}} - s^* C(X) = \{ \emptyset, X, \{a\}, \{c\}, \{a, b\} \}.$

Therefore, $S_{\tau_{12}}$ - $s^*C(X) \neq S_{\tau_{21}}$ - $s^*C(X)$.

Proposition 3.18 If $S_{\tau_1} \subseteq S_{\tau_2}$ in $(X, S_{\tau_1}, S_{\tau_2})$ then $S_{\tau_{21}} - s^*C(X) \subseteq S_{\tau_{12}} - s^*C(X)$.

Proof: Let A be $S_{\tau_{21}}$ - $s^*C(X)$. Then S_{τ_1} - $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is $S_{\tau_2} = g^*$ -open. Since $S_{\tau_1} \subseteq S_{\tau_2}$, S_{τ_2} - $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is S_{τ_1} - g^* - open. Hence A is $S_{\tau_{12}}$ - $s^*C(X)$.

Remark 3.19 The converse of the above Proposition 3.18, is not true from the following example.

Example 3.20 *Let* $X = \{a, b, c, d\}$,

 $S_{\tau_1} = \{\emptyset, X, \{a, b\}\},\$

 $S_{\tau_2} = \{ \emptyset, X, \{a, b, d\} \},\$

 $S_{\tau_{12}}-s^*C(X) = \{\emptyset, X, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

 $S_{\tau_{21}}-s^*C(X) = \{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}.$

Then $S_{\tau_{21}}$ -s* $C(X) \subseteq S_{\tau_{12}}$ -s*C(X) but S_{τ_1} is not contained in S_{τ_2} .

Proposition 3.21 If A is $S_{\tau_{ii}}$ -sag*-closed, then S_{τ_i} -acl(A)- A contains no non empty S_{τ_i} -g*-closed set.

Proof: Let A be $S_{\tau_{ij}}$ -sag*-closed set and F be a non empty S_{τ_i} -g*-closed subset such that $F \subseteq S_{\tau_j}$ -acl(A)– $A = S_{\tau_j}$ -acl(A) $\subseteq A^c$. Therefore, $F \subseteq S_{\tau_i}$ -acl(A) and $F \subseteq A^c$.

Since F^c is a S_{τ_i} - g^* -open and A is $S_{\tau_{ij}}$ - sag^* -closed set in $(X, S_{\tau_1}, S_{\tau_2})$, We have S_{τ_j} - $acl(A) \subseteq F^c$. That is $F \subseteq [S_{\tau_i} - cl(A)]^c$. Hence $F \subseteq [S_{\tau_i} - cl(A)] \cap [S_{\tau_i} - cl(A)]^c = \emptyset$. Therefore, $F = \emptyset$. Hence, S_{τ_j} -acl(A)- A contains no non empty $S_{\tau_i}g^*$ -closed set.

Remark 3.22 The converse of the above Proposition 3.21, is not true as seen from the following example.

Example 3.23 In Example 3.11,

 $S_{\tau_{12}}-s^*C(X) = \{\emptyset, X, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\}.$

If $A = \{b, c, d\}$ then S_{τ_2} - $acl(A) - A = \{a\}$ does not contain any non empty S_{τ_2} - g^* -closed set. But A is not a $S_{\tau_{ij}}$ -sag*-closed set.

Corollary 3.24 If A is $S_{\tau_{ij}}$ -sag*-closed set in $(X, S_{\tau_1}, S_{\tau_2})$, then A is S_{τ_j} - α - closed if and only if S_{τ_j} - $\alpha cl(A)$ - A is S_{τ_i} - g^* -closed.

Proof:

Necessity:

If A is S_{τ_i} - α -closed, then S_{τ_i} - α cl(A) = A. That is S_{τ_i} -cl(A) - A = \emptyset and hence S_{τ_i} - α cl(A) - A is S_{τ_i} -g*-closed.



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Sufficiency:

If S_{τ_j} - $\alpha cl(A) - A$ is S_{τ_i} - g^* -closed, then by Proposition 3.21, S_{τ_j} - $\alpha cl(A) - A = \emptyset$, since A is $S_{\tau_{ij}}$ - $s\alpha g^*$ -closed. Therefore A is S_{τ_i} - α -closed.

Proposition 3.25 If A is an S_{τ_j} -sag^{*}-closed set of $(X, S_{\tau_1}, S_{\tau_2})$ such that $A \subseteq B \subseteq S_{\tau_j}$ -acl(A), then B is also an $S_{\tau_{ij}}$ -sag^{*}-closed set of $(X, S_{\tau_1}, S_{\tau_2})$, where i, j = 1, 2 and $i \neq j$.

Proof: Let U be a S_{τ_i} - g^* -open set of $(X, S_{\tau_1}, S_{\tau_2})$, such that $B \subseteq U$. Then, $A \subseteq U$. Since A is a $S_{\tau_{ij}}$ - sag^* -closed set, then S_{τ_j} - $acl(A) \subseteq U$. Also, S_{τ_j} - $acl(B) \subseteq S_{\tau_j}$ - $acl(S_{\tau_j}$ - $acl(A)) = S_{\tau_j}$ - $acl(A) \subseteq U$ which implies, S_{τ_j} - $acl(B) \subseteq U$. Hence, B is also an $S_{\tau_{ij}}$ - sag^* -closed set of $(X, S_{\tau_1}, S_{\tau_2})$.

Remark 3.26 Intersection of $S_{\tau_{ij}}$ -sag*-closed sets and S_{τ_i} -g*-open sets is neither $S_{\tau_{ij}}$ -sag*-closed nor S_{τ_i} -g*-open as can be seen from the following example.

Example 3.27 Let S_{τ_1} and S_{τ_2} in Example 3.11,

 $S_{\tau_1} - g^* O(X) = \{ \emptyset, X, \{a\}, \{d\}, \{a, d\}, \{b, c, d\} \},\$

 $S_{\tau_1} - s^* C(X) = \{\emptyset, X, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\}.$ We have $A = \{b, c, d\}$ is $S_{\tau_1} - g^*$ -open and $B = \{a, b, c\}$ is $S_{\tau_{12}}$ -sag*-closed set, their intersection $\{b, c\}$ is neither $S_{\tau_{12}}$ -sag*-closed set nor $S_{\tau_1} - g^*$ -open set.

Proposition 3.28 For each element $x \in X$, the singleton $\{x\}$ is S_{τ_i} - g^* closed or $\{x\}^c$ is $S_{\tau_{ij}}$ -sag*- closed.

Proof: Suppose that $\{x\}$ is not S_{τ_i} - g^* -closed. Then $\{x\}^c$ is not S_{τ_i} - g^* -open and X is the only S_{τ_i} - g^* -open set which contains $\{x\}^c$ and $\{x\}^c$ is $S_{\tau_{ii}}$ -sag*-closed.

Proposition 3.29 In a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ if S_{τ_i} -g*-open $\subseteq S_{\tau_j}$ -closed then every subset of X is an $S_{\tau_{ij}}$ -sag*-closed set of $(X, S_{\tau_1}, S_{\tau_2})$.

Proof: Let S_{τ_i} - g^* -open $\subseteq S_{\tau_i}$ -closed. Let A be a subset of X such that $A \subseteq U$, Where U is S_{τ_i} - g^* -open set. Then S_{τ_j} - $\alpha cl(A) \subseteq S_{\tau_i}$ -cl(A) $\subseteq S_{\tau_i}$ -cl(U) and hence A is an $S_{\tau_{ij}}$ -sag*-closed set of $(X, S_{\tau_1}, S_{\tau_2})$.

IV. $S_{\tau_{ii}}$ -STRONGLY αg^* -OPEN SETS

In this section we introduce the concept of $S_{\tau_{ii}}$ -strongly αg^* -open sets in supra bitopological spaces.

Definition 4.1 A subset A of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is said to be supra τ_{ij} -stronglyag*-open(briefly, $S_{\tau_{ij}}$ -sag*-open) set if A^c is $S_{\tau_{ij}}$ -sag*-closed set in $(X, S_{\tau_1}, S_{\tau_2})$.

The family of all $S_{\tau_{1i}}$ -sag*-open sets of $(X, S_{\tau_1}, S_{\tau_2})$ is denoted by $S_{\tau_{1i}}$ -s*O(X).

Remark 4.2 The union of any two $S_{\tau_{ij}}$ -sag*-open set is not necessary $S_{\tau_{ij}}$ -sag*open set as seen in the following example.

Example 4.3 *Let* $X = \{a, b, c, d\}$,

 $S_{\tau_1} = \{\emptyset, X, \{b\}, \{b, c\}, \{a, c, d\}\},\$

 $S_{\tau_2} = \{\emptyset, X, \{a\}, \{a, d\}, \{b, c, d\}\},\$

 $S_{\tau_{12}}$ -s* $O(X) = \{\emptyset, X, \{a\}, \{c\}, \{d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Here $\{a\}$ and $\{d\}$ are $S_{\tau_{12}}$ -sag*-open sets but their union $\{a\} \cup \{c\} = \{a, c\}$ is not $S_{\tau_{12}}$ -sag*- open.



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Remark 4.4 The intersection of any two $S_{\tau_{ij}}$ -sag*-open set is not necessary $S_{\tau_{ij}}$ -ag*-open set as in the following example.

Example 4.5 In Example 4.3,

 $S_{\tau_{12}}-s^*O(X) = \{\emptyset, X, \{a\}, \{c\}, \{d\}, \{c,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}. Here \{b,c,d\} and \{a,b,d\} are S_{\tau_{12}}-sag^*-open sets but \{b,c,d\} \cap \{a,b,d\} = \{b,d\} is not S_{\tau_{12}}-sag^*-open.$

Remark 4.6 Every S_{τ_1} -open set is $S_{\tau_{ij}}$ -sag*-open but the converse is not true in general as can be seen from the following example.

Example 4.7 *Let* $X = \{a, b, c\}$,

 $S_{\tau_1} = \{\emptyset, X, \{a, b\}, \{b, c\}\},\$

 $S_{\tau_2} = \{ \emptyset, X, \{c\} \{a, b\} \},\$

 $S_{\tau_{12}}\text{-}s^*O(X)=\{\emptyset,X,\{b\},\{c\},\{a,b\},\{b,c\}\}.$

Here $\{b\}$ *is* $S_{\tau_{12}}$ *-sag*-open but it is not* S_{τ_1} *- open.*

Remark 4.8 The intersection of $S_{\tau_{ij}}$ -sag*-open set and S_{τ_j} -open set is not $S_{\tau_{ij}}$ -sag*-open as seen from the following example.

Example 4.9 In Example 4.3,

 $S_{\tau_{12}}-s^*O(X)=\{\emptyset,X,\{a\},\{c\},\{d\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}.$

Here $A = \{a,d\}$ is S_{τ_2} -open and $B = \{a,b,d\}$ is $S_{\tau_{12}}$ -sag*-open but their intersection $\{a,d\}$ is not $S_{\tau_{12}}$ -sag*- open.

Theorem 4.10 A subset of supra bitopological space in $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$ -sag*open set if and only if $B \subseteq S_{\tau_j}$ -aint(A) whenever B is S_{τ_i} -g*-closed and $B \subseteq A$, where i, j = 1, 2 and $i \neq j$.

Proof: Let A be τ_{ij} -sag^{*}-open set. Let B be a S_{τ_i} -g^{*}-closed set such that $B \subseteq A$. Let $A \subseteq B$ and B is S_{τ_i} -g^{*}-closed. Then $A^c \subseteq B^c$ and B^c is S_{τ_i} -g^{*}-open. We have A^c is $S_{\tau_{ij}}$ -sag^{*}-closed. Hence, $[S_{\tau_j} - acl(A)^c] \subseteq B^c$. Therefore, $B \subseteq S_{\tau_j}$ -aint(A). Conversely, suppose $B \subseteq S_{\tau_j}$ -aint(A) whenever $B \subseteq A$ and B is S_{τ_i} -g^{*}-closed. Let $A^c \subseteq U$ and U is S_{τ_i} -g^{*}-open. Then $U^c \subseteq A$ and U^c is S_{τ_i} -g^{*}-closed. By hypothesis $U^c \subseteq S_{\tau_j}$ -aint(A). Hence, $[S_{\tau_j} - aint(A)]^c \subseteq U$. That is $[S_{\tau_j} - acl(A)^c] \subseteq U$. Consequently, A^c is $S_{\tau_{ij}}$ -sag^{*}-closed set. Hence, A is $S_{\tau_{ij}}$ -sag^{*}-open.

Theorem 4.11 Let A and B be subsets of supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ such that S_{τ_j} -aint(A) $\subseteq B \subseteq A$. If A is $S_{\tau_{ij}}$ -sag*-open, then B is $S_{\tau_{ij}}$ -sag*- open, where i, j = 1, 2 and $i \neq j$.

Proof: Let A be $S_{\tau_{ij}}$ -sag*-open. Let U be a S_{τ_i} -g*-closed such that $U \subseteq B$. Since $U \subseteq B$ and $B \subseteq A$, we have $U \subseteq A$. Therefore, $U \subseteq S_{\tau_j}$ -aint(A). Since S_{τ_j} -aint(A) $\subseteq B$, we have S_{τ_j} -aint(S_{τ_j} -aint(A)) $\subseteq S_{\tau_j}$ -aint(B). Therefore, S_{τ_j} -aint(A) int(A)

 $\subseteq S_{\tau_i}$ - $\alpha(B)$. Consequently, $U \subseteq S_{\tau_i}$ - $\alpha(B)$. Hence, B is $S_{\tau_{ii}}$ -sag*-open

Theorem 4.12 A subset A is $S_{\tau_{ii}}$ -sag*-closed set then S_{τ_i} -acl(A) – A is $S_{\tau_{ii}}$ -sag*- open.

Proof: Let A be $S_{\tau_{ij}}$ -sag*-closed set. Let $F \subseteq S_{\tau_j}$ -acl(A) – A, where F is S_{τ_i} -g*-closed set. Since A is $S_{\tau_{ij}}$ -sag*-closed, we have S_{τ_j} -acl(A) – A does not contain non empty S_{τ_i} -g*-closed by Proposition 3.21, Consequently, F = Ø. Therefore, $\emptyset \subseteq S_{\tau_j}$ -acl(A) – A, $\emptyset \subseteq S_{\tau_j}$ -acl(A) – A), we obtain $F \subseteq S_{\tau_j}$ -acl(A) – A).



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Hence, S_{τ_i} - $\alpha cl(A) - A$ is $S_{\tau_{ii}}$ - $s\alpha g^*$ -open.

Remark 4.13 From the above results, we get the following relations.

$$S_{\tau_{ij}} - closed$$

$$S_{\tau_{ij}} - g^* - closed \rightarrow S_{\tau_{ij}} - sag^* - closed$$

The converse of the above relations is not true.

V. $S_{\tau_{ii}}$ * T_c , $S_{\tau_{ii}}$ * T_g AND $S_{\tau_{ii}}$ * T_g *-SPACES IN SUPRA BITOPOLOGICAL SPACES

In this section, We introduce $S_{\tau_{ij}} \cdot *T_c$ -space, $S_{\tau_{ij}} \cdot *T_g$ -space and $S_{\tau_{ij}} \cdot *T_g$ *- space in supra bitopological spaces with the help of $S_{\tau_{ij}} \cdot sag^*$ -closed set.

Definition 5.1 A supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is said to be $S_{\tau_{ij}} \cdot s T_c$ - space if every $S_{\tau_{ij}} \cdot s \alpha g^*$ -closed set is S_{τ_i} -closed.

Proposition 5.2 If $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$ -space, then it is an $S_{\tau_{ij}}$ - $T_{1/2}^*$ -space but not conversely.

Example 5.3*Let* $X = \{a, b, c\},\$

 $S_{\tau_1} = \{\emptyset, X, \{c\}\},\$

 $S_{\tau_2} = \{\emptyset, X, \{c\}, \{a, b\}\}.$

Then $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$ - $T^*_{1/2}$ -space but not a $S_{\tau_{ij}}$ -s+ T_c - space.

Theorem 5.4*A supra bitopological space* $(X, S_{\tau_1}, S_{\tau_2})$ *is* $S_{\tau_{ij}}$ -*s* $*T_c$ -*space if and only if* $\{x\}$ *is* S_{τ_j} -*open or* S_{τ_i} -*g**-*closed for each* $x \in X$.

Proof: Suppose that $\{x\}$ is not S_{τ_i} - g^* -closed. Then by Proposition 3.28, $\{x\}^c$ is $S_{\tau_{ij}}$ - $s\alpha g^*$ -closed set. Since $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$ - s^*T_c -space, $\{x\}^c$ is S_{τ_j} - closed. Therefore $\{x\}$ is S_{τ_j} - open.

Conversely, let A be an $S_{\tau_{ij}}$ -sag*-closed set. By assumption, $\{x\}$ is S_{τ_j} -open or S_{τ_i} -g*-closed. for any $x \in S_{\tau_j}$ -acl(A) $\subseteq S_{\tau_i}$ - cl(A).

Case (i): Suppose that $\{x\}$ is S_{τ_i} -open. Since $\{x\} \cap A \neq \emptyset$, We have $x \in A$.

Case (ii): Suppose that $\{x\}$ is S_{τ_j} - g^* -closed. If x not in A, then $\{x\}\subseteq S_{\tau_j}$ - $\alpha cl(A) - A$, Which is a contradiction to Proposition 3.21. Therefore $x \in A$.

Thus in both cases, We conclude that A is S_{τ_i} -closed. Hence $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_i} * Tc$ -space.

Remark 5.5(X, S_{τ_1})-space is not generally a S_{τ} - $_s*T_c$ -space even if $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{12}}$ - $_s*T_c$ -space as shown in the following example.

Example 5.6*Let* $X = \{a, b, c\},$ $S_{\tau_1} = \{\emptyset, X, \{c\}, \{a, b\}, \{b, c\}\},$ $S_{\tau_2} = \{\emptyset, X, \{b\}, \{a, c\}, \{a, c\}, \{b, c\}\}.$

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Then (X, S_{τ_1}) is not a $S_{\tau^{-s}} * T_c$ -space but $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{12}} - s * T_c$ -space.

Definition 5.7*A supra bitopological space* $(X, S_{\tau_1}, S_{\tau_2})$ *is said to be pairwise* S_{τ} *-s** T_c *-space if it is both* $S_{\tau_{12}}$ *-s** T_c *and* $S_{\tau_{21}}$ *-s** T_c *-space.*

Proposition 5.8 If $(X, S_{\tau_1}, S_{\tau_2})$ is pairwise S_{τ} -space then it is pairwise S_{τ} - $T_{1/2}^*$ -space but not conversely.

Example 5.9 In Example 5.3, Then $(X, S_{\tau_1}, S_{\tau_2})$ is pairwise S_{τ} - $T_{1/2}^*$ -space. But

 $(X, S_{\tau_1}, S_{\tau_2})$ is not a pairwise S_{τ} -s* T_c -space, since it is not a $S_{\tau_{21}}$ -s* T_c -space.

Definition 5.10*A supra bitopological space* $(X, S_{\tau_1}, S_{\tau_2})$ *is said to be* $S_{\tau_{ij}}$ -*s** T_g *space if every* $S_{\tau_{ij}}$ -*sag**-*closed set is* $S_{\tau_{ij}}$ -*g*-*closed.*

Proposition 5.11 If $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ii}}$ -s* T_c -space is an $S_{\tau_{ii}}$ -s* T_g -space but not conversely.

Example 5.12*Let* $X = \{a, b, c\},\$

 $S_{\tau_1} = \{\emptyset, X, \{c\}, \{a, b\}, \{b, c\}\},\$

 $S_{\tau_2} = \{ \emptyset, X, \{a, c\}, \{b, c\} \}.$

Then $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{12}}$ -s* T_g -space but not a $S_{\tau_{12}}$ -s* T_c -space.

Theorem 5.13*A supra bitopological space* $(X, S_{\tau_1}, S_{\tau_2})$ *is* $S_{\tau_{ij}}$ -s* T_c -space if and only if it is both $S_{\tau_{ij}}$ -s* T_g and $S_{\tau_{ij}}$ - $T_{1/2}^*$ -space.

Proof: Suppose that $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ii}}$ -space. Then by Proposition 5.2 and

Proposition 5.11($X, S_{\tau_1}, S_{\tau_2}$) is $S_{\tau_{ij}} - T^*_{1/2}$ -space and $S_{\tau_{ij}} - space$.

Conversely, suppose that $(X, S_{\tau_1}, S_{\tau_2})$ is both $S_{\tau_{ij}}$ -space and $S_{\tau_{ij}}$ - $T_{1/2}^*$ space.

Let A be a $S_{\tau_{ij}}$ -sag*-closed set of $(X, S_{\tau_1}, S_{\tau_2})$. since $(X, S_{\tau_1}, S_{\tau_2})$ is an $S_{\tau_{ij}}$ -s* T_g space, A is an $S_{\tau_{ij}}$ -g-closed set. Since $(X, S_{\tau_1}, S_{\tau_2})$ is an $S_{\tau_{ij}}$ - $T_{1/2}^*$ -space, A is S_{τ_i} -closed set of $(X, S_{\tau_1}, S_{\tau_2})$. Therefore, $(X, S_{\tau_1}, S_{\tau_2})$ is an $S_{\tau_{ij}}$ -s* T_c -space.

Definition 5.14*A supra bitopological space* $(X, S_{\tau_1}, S_{\tau_2})$ *is said to be* $S_{\tau_{ij}}$ -*s** T_s *-*space if every* $S_{\tau_{ij}}$ -*sag**-*closed set is* $S_{\tau_{ij}}$ -*g**-*closed set.*

Proposition 5.15 *Every* $S_{\tau_{ii}}$ -s+ T_g *-space is $aS_{\tau_{ii}}$ -s+ T_g -space but not conversely.

Example 5.16*Let* $X = \{a, b, c\},\$

 $S_{\tau_1} = \{ \emptyset, X, \{b, \} \},\$

 $S_{\tau_2} = \{\emptyset, X, \{b, c\}\}.$

Then $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{12}}$ -s* T_g -space but not a $S_{\tau_{12}}$ -s* T_g *- space.

Proposition 5.17*Every* $S_{\tau_{ij}}$ -s* T_c -space is an $S_{\tau_{ij}}$ -s* T_g *-space but not conversely from the following example.

Example 5.18Let $(X, S_{\tau_1}, S_{\tau_2})$ be as in Example 5.3. Then $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{12}}$ -s* T_g *-space but not a $S_{\tau_{12}}$ -s* T_c - space.

Theorem 5.19*A supra bitopological space* $(X, S_{\tau_1}, S_{\tau_2})$ *is* $S_{\tau_{ij}}$ -*space if and only if it is both* $S_{\tau_{ij}}$ -*s* *T_g **and* $S_{\tau_{ij}}$ - $T_{1/2}^*$ -*space* $X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$ - $T_{1/2}^*$



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Proof: Suppose that $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_i j^- s} * T_c$ -space. Then by Proposition 5.2 and

Proposition 5.17, $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$ - $T_{1/2}^*$ -space and $S_{\tau_{ij}}$ - $s*T_g*$ - space.

Conversely, suppose that $(X, S_{\tau_1}, S_{\tau_2})$ is both $S_{\tau_{ij}}$ - $s*T_g*$ -space and $S_{\tau_{ij}}$ - $T^*_{1/2}$ -space.

Let A be a $S_{\tau_{ij}}$ -sag*-closed set of $(X, S_{\tau_1}, S_{\tau_2})$. Since $(X, S_{\tau_1}, S_{\tau_2})$ is an $S_{\tau_{ij}}$ -s* T_g *-space, A is an $S_{\tau_{ij}}$ -g*-closed set. Since $(X, S_{\tau_1}, S_{\tau_2})$ is an $S_{\tau_{ij}}$ - $T_{1/2}^*$ -space, A is S_{τ_i} -closed set of $(X, S_{\tau_1}, S_{\tau_2})$. Therefore, $(X, S_{\tau_1}, S_{\tau_2})$ is an $S_{\tau_{ij}}$ -s* T_c -space.

Remark 5.20From the above results we have the following implications.

$$S_{\tau_{ij}}$$
-s T_c -space $\rightarrow S_{\tau_{ij}}$ - $T_{1/2}^*$ -space.

$$\sim$$

 $S_{\tau_{ii}}$ -s* T_g *-space $\rightarrow S_{\tau_{ii}}$ -s* T_g -space

The converse of the above implications is not true.

VI. CONCLUSION

In this paper, we have introduced the new class of sets called $S_{\tau_{ij}}$ -sag*-closed sets, $S_{\tau_{ij}}$ -sag*-open sets and compared their results of $S_{\tau_{ij}}$ -g*-closed sets. Also, we construct three new types of supra bitopological spaces. Further, we are interested to find some interesting results in supra bitopological spaces.

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